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THE EFFECTIVENESS OF DISCRIMINANT MODELS BASED ON THE EXAMPLE OF THE MANUFACTURING SECTOR

The best models of bankruptcy prediction have been selected that can indicate the deteriorating situation of a company several years before bankruptcy occurs. There are a lot of methods for evaluating the financial statements of enterprises, but only a few can assess a company as a whole and recognise sufficiently early the deteriorating financial standing of a business. The matrix method was used to classify companies in order to assess the models. The correctness of the classification made by the models was tested based on data covering a period of five years before the bankruptcy of enterprises. To analyse the effectiveness of these discriminant models, the financial reports of manufacturing companies were used. Analysis of 33 models of bankruptcy prediction shows that only 5 models were characterized by sufficient predictive ability in the five years before the bankruptcy of enterprises. The results obtained show that so far a unique, accurate, optimal model, by which companies could be assessed with very high efficiency, has not been identified. That is why it is vital to continue research related to the construction of models enabling accurate evaluation of the financial condition of businesses.

Keywords: *insolvency, models of bankruptcy prediction, manufacturing sector*

1. Introduction

The purpose of this article was to select the best discriminant models that indicate the deteriorating situation of a given business several years prior to bankruptcy. For this reason, we conducted a comprehensive survey to assess the large number of discriminant models that presently exist.

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The research objective formulated in this way will allow us to address the following research questions:

• Have the models employed to assess the financial condition of firms correctly and sufficiently early signalled the deteriorating financial state of a firm within the manufacturing sector?

• Has the overall efficiency of models constructed at the end of the previous century decreased significantly (due, e.g., to changes in the market and financial management)?

• Do the number of indicators incorporated into a particular model influence its overall efficiency?

• Do models designed specifically for a particular sector assess firms more accurately than universal ones (not specifically designed for a particular sector)?

Our survey differs from previous surveys [17, 27, 42, 45, 51, 52, 56, 75, 76, 79, 81, 84, 91, 95–98] in a number of dimensions. First, the scope of our survey is very broad. We examined 33 discriminant models. Second, we sampled a large cross-section of approximately 11 000 enterprises. Third, we analysed the manufacturing sector. Since 2004, the majority of companies in the Polish manufacturing sector have gone bankrupt [94].

2. Literature review

There are many discriminant models. The most famous one is Altman's synthetic model [4] which is based on five financial ratios. Altman was the initiator of using multidimensional analysis to predict corporate bankruptcy. In the following years, others began to create discriminant analysis models taking inspiration from Altman's model. The most important approaches include: Altman [9, 11], Blum [21], Chang, Afifi [22], Deakin [25, 26], Edmister [34], Fulmer et al. [37], Libby [64], Karels, Prakash [53], Ketz [54], Koh, Killough [58], Meyer, Pifer [68], Moses, Liao [70], Pettway, Sinkey Jr. [74], Sinkey Jr. [83].

Work on creating predictive models of business failure was also carried out outside the United States. To name a few, such models were developed in: China (Zhang et al. [108]), Finland (Laitinen [61]), France (Altman et al. [13], Bardos [18], Micha [69]), Japan (Ko [57]), Canada (Legault [62], Springate [86]), Korea (Altman et al. [10]), Germany (Weinrich [102]), United Kingdom (El Hennway, Morris [35], Goudie, Meeks [43], Peel [73], Taffler [88]), Italy (Altman et al. [12], Appetiti [16]).

In Poland after the political transformation, many models have been designed to foresee the upcoming bankruptcy of a company. The most popular models are the following: Appenzeller (Hadasik) [14, 15], Dominiak, Mazurkiewicz [31], Hamrol et al. [46], Hołda [48], Gajdka, Stos [38–40], Juszczyk, Balina [52], Mączyńska [66, 67], Pogodzińska, Sojak [77], Prusak [79], Wierzba [103], Wiśniewska [104], Zięba et al. [110].

So far, many scientists from various research centres have undertaken the verification of the efficiency of existing models for predicting the bankruptcy of enterprises. In the worldwide literature, one regularly comes across verification of the Altman model. In the study conducted by Altman, one can see that his model was characterized by a high rate of correct classification of companies in the year before bankruptcy – more than 90% accurate in individual studies. However, the recognition of companies which go bankrupt does not occur at such a high level.

Grice and Ingram [44] also investigated the effectiveness of the Altman model over time. They stated that a model based on data from a specific period is characterized by a high accuracy in classifying enterprises only at that time. As time passes, the accuracy of classification based on such a model decreases. It is worth noting that Grice and Ingram studied the model on a much larger sample of companies than Altman, which means that the results of their research can be regarded as more reliable. In addition to the research conducted by Grice and Ingram, other researchers have also undertaken validation of the Altman model and other models based on data from outside the USA [2, 3, 23, 28–30, 32, 63, 65, 89, 99]. The model of Altman has not only been verified, but has also been modified and specially adapted to different countries in various ways. Such studies have been carried out, among others, by Altman et al. [8], Begley et al. [19], Ho et al. [47], Huo [49], Kwak et al. [60], Wang, Campbell [100, 101], Xu, Zhang [107].

Similarly in Poland, such verification of models has been carried out by, among others: Adamowicz and Noga [1], Balina [17], Dec [27], Juszczyk, Balina [51, 52], Gołębiewski, Żywno [42], Hamrol, Chodakowski [45], Kisielińska [55, 56], Pieńkowska [75, 76], Rogowski, Duleba [80], Rusek [81], Sobuhoro, Stefański [84], Tymoczuk [98], Tomczak [91, 95].

The results of Polish research vary according to both the period and the size of the test sample used for verification. The period covered by the data is very important, because one of the disadvantages of such models is the decrease in their efficiency with the passage of time, since a model is adapted to the data it is based on. Similarly, in this case, the accuracy of classification declines with time. Particularly from analysing the study of Balina, Juszczyk and Balina, one can note a significant decrease in the forecasting power of models. In turn, Tymoszuk examined the financial condition of enterprises in the two years before bankruptcy. The main conclusion of the study is the following: if one wants to use models of bankruptcy prediction to assess the financial condition of enterprises, one should use those models which are based on the most recent data [98]. It is worth pointing out that the highest efficiency was achieved by the INE Pan7 model and the lowest by the Hołda model. Furthermore, the Hołda model did not possess high predictive ability in other studies [41]. It should also be noted that the verification of models was based on small study samples (up to a maximum of 203 enterprises), which may underestimate the efficiency of models. Sometimes, different models of bankruptcy prediction indicate that a given firm is in different financial situations [24, 71, 85, 106]. Therefore, it is advised to use a number of various models in order to assess a company's financial condition [55, 87].

It is worth emphasizing that verification of these models has been carried out both by their authors and other researchers. Generally, the verification of models has been based only on a small study sample. In many cases, the test sample did not exceed a hundred enterprises. According to the authors, such a small research sample does not sufficiently show the efficiency of a particular model. Given this fact, models will be assessed on the basis of research samples of various sizes, in order to get a complete picture of the performance of models.

3. Dataset

The financial statements of businesses were drawn from the EMIS database (EMIS stands for Emerging Markets Information Service, a Euromoney Institutional Investor Company, www.emis.com). At the time of selecting companies in the EMIS base, there were over seventeen thousand still in operation, seven thousand of which did not meet the criterion for including them in the sample (the criterion for selecting a firm was the availability of at least three consecutive financial reports in the period 2000–2012). Moreover, there were also a thousand companies which had filed for bankruptcy (these cases were verified using the National Court Register). Three hundred of these bankrupt firms did not meet the criterion for inclusion, which required at least one financial report to be available in the five-year period before filing for bankruptcy. The small and big samples were chosen from among these approximately seven hundred bankrupt firms and over ten thousand companies remaining in operation. In this paper, the notion of "one year before filing for bankruptcy" is not a crisp concept, since the last financial report is not given exactly one year before filing for bankruptcy. In this case, a year before filing for bankruptcy means between nine months and fifteen months before filing for bankruptcy.

The small sample consists of 424 companies (limited liability companies, joint stock companies, partnerships and cooperatives). One half of them (212 companies) had gone bankrupt in the period 2008–2013 (since the outbreak of the crisis) and the other half remained in operation (in good financial standing). The firms remaining in operation were selected to be similar to (paired with) the insolvent companies. In the process of selecting these companies, the following criteria were used: industry and the reporting period, as well as the financial condition and size of the company.

The large study sample consists of nearly 600 enterprises that went bankrupt in the period of 2007–2013 (almost 2100 financial statements were analysed) and the more than 10 000 businesses still functioning (in this trial other companies, which declared bankruptcy were excluded). The data covered the period 2000–2012 (more than 65 thousand financial statements were taken into consideration). Overall, a total of nearly 700 bankrupt firms and over 10,000 still operating companies have been used for analysis. This figure corresponds to the amount of data available.

The large sample consists of many branches of the manufacturing sector, which are divided according to the NAICS system: nonmetallic mineral product manufacturing (742 companies), fabricated metal product manufacturing (1218), printing and related support activities (335), food manufacturing (1564), petroleum and coal products manufacturing (46), textile product mills (160), computer and electronic product manufacturing (455), machinery manufacturing (1006), furniture and related product manufacturing (482), beverage and tobacco product manufacturing (163), apparel manufacturing (363), paper manufacturing (342), transportation equipment manufacturing (485), miscellaneous manufacturing (191), electrical equipment, appliance, and component manufacturing (422), wood product manufacturing (486), chemical manufacturing (670), plastics and rubber products manufacturing (1040), leather and allied product manufacturing (94), primary metal manufacturing (256), textile mills (82). The sample covers both large, medium-sized and small businesses. All of the companies operated in Poland.

4. Methodology of research

The aim of the study was to select the most effective models that indicate the declining financial standing of manufacturing businesses several years before insolvency is declared. Therefore, thirty three discriminant models: Altman [5], Appenzeller, Szarzec 1, 2 [15], Gajdka, Stos 1–4 [38–40], Hadasik 1–5 [14], Hołda [48], INE PAN 1–7 [67], Janek, Żuchowski [50], Legault, Score [62], quick test; Mączyńska [66], Pogodzińska, Sojak [77], Poznan [46], Prusak 1–4 [79], Springate [86], Taffler [88], Wierzba [103] were compared and rated by analysing both a small study sample and a large study sample.

In order to compare models, the classification matrix method was chosen. This method is the most frequently used to compare and evaluate existing models [1, 4, 55, 67, 79][2]. Such analysis enables assessment of the two types (I and II) of efficiency of models over time and thus identifies a list of models that evaluate the financial condition of enterprises in the most accurate way. Type I efficiency (EI) is the percentage of properly classified bankrupt companies, while type II efficiency (EII) is the percentage of non-bankrupt enterprises correctly classified. The correct diagnosis of a company's health is critically decisive, because of its consequences and costs. Analysis of type I efficiency is based on nearly 600 insolvent companies in the period 2007–2013, whereas the analysis of type II efficiency is based on the more than 10 000 companies still operating (this sample excluded some of the companies which declared bankruptcy in the period 2000–2012).

[2]This is not the only method to compare models; another method is the ROC curve [82, 105].

A range of attempts have been carried out to estimate the difference between the error rates of type I and type II for a pair of models. The type I error rate is the percentage of misclassified bankrupt businesses, while the type II error rate is the percentage of still operating companies incorrectly classified. It is worth emphasizing that an error of type I is much more expensive for a bank than an error of type II in terms of a decision regarding whether to grant a loan to a company or not. An error of type I occurs when the bank gives a loan to an enterprise which is not able to pay it back, whereas an error of type II involves only the loss of the benefits that the bank would receive if the company repaid a loan, for example loss of interest on the loan. The ratio of the type I error rates to type II error rates is of the order 31:1 [5], 35:1 [6], 38:1 [36], 41:1 [72]. In contrast, the relative costs of making these two mistakes is 58:1 [109]. Consequently, it is reasonable to place more stress on minimizing the type I error rate of a model.

The efficiency of the models was tested over a five-year period before the bankruptcy of enterprises. Models were ranked according to the highest average value of their overall performance in the five years prior to their bankruptcy.

5. Results

Tables 1–5 show the top 5 models for classifying businesses according to various criteria. In these tables, models are ranked according to the aggregate mean values of their performance in each set of trials. Four of them (Prusak 4, Prusak 3, Legault Score, and Springate) have been designed for manufacturing companies. The results presented in these tables will help to provide answers to the research questions.

Table 1. The efficiency of selected models for the small research sample (424 companies) [%]

Period before bankruptcy	5 years	4 years	3 years	2 years	1 year	A1–5	A1–5
Model	EI	EI	EI	EI	EI	EI	TE
	EII	EII	EII	EII	EII	EII	
Prusak 4	67.4	64.7	68.8	73.0	79.2	70.6	75.7
	76.5	78.4	82.6	82.1	84.0	80.7	
Gajdka, Stos 2	65.8	65.8	65.7	69.5	75.0	68.3	75.5
	72.8	81.4	84.9	86.3	87.8	82.6	
Prusak 3	52.8	54.6	62.9	68.6	77.2	63.2	73.9
	82.7	81.4	84.9	85.8	88.2	84.6	
Legault, Score	42.8	42.3	42.5	50.5	63.0	48.2	73.0
	94.5	99.2	97.9	98.6	99.1	97.8	
Springate	39.1	45.3	53.7	57.7	73.3	53.8	72.9
	93.1	89.3	91.6	92.9	92.9	92.0	

TE – total efficiency, A1–5 – five year average. Source: Authors' work.

Analysing Table 1, which presents the type I and type II efficiencies for the small sample, it should be noted that the highest overall type I efficiency for correctly classifying bankrupt companies is achieved by the Prusak 4 model – a type I efficiency of almost 71% over the five years before bankruptcy. However, the highest overall type II efficiency for correctly classifying operating firms is achieved by the Legault–Score model. However, this model recognizes businesses that go bankrupt very poorly and is designed for companies operating in Canada. Given these facts, the results obtained using this model should only be taken into consideration as a basis for comparison with the results of Polish models.

Table 2. The type I efficiency [%] of selected models
for the large research sample (600 companies)

Period before bankruptcy	5 years	4 years	3 years	2 years	1 years	Average1–5
Model	EI	EI	EI	EI	EI	TE
Prusak 4	66.8	67.3	76.1	75.6	78.1	72.8
Gajdka, Stos 2	61.8	62.6	66.7	65.1	74.1	66.1
Prusak 3	53.5	58.6	65.5	69.8	72.1	63.9
Springate	52.2	56.8	64.0	68.4	72.4	62.7
Legault, Score	51.7	52.4	57.4	58.2	60.5	56.0

Source: Authors' work.

Table 3. The type II efficiency of selected models
for the large research sample (10 000 companies) [%]

Model	2000	…	2009	2010	2011	2012	Average (2000–2012)
Legault,Score	57.2	…	72.9	71.3	73.8	64.2	67.3
Gajdka, Stos 2	62.2	…	61.5	61.5	66.3	52.1	64.8
Prusak 3	58.3	…	60.5	57.5	62.8	57.1	61.7
Springate	45.2	…	62.6	62.2	64.3	63.4	57.6
Prusak 4	48.1	…	53.1	49.4	53.9	47.5	53.0

Source: Authors' work.

Table 4. The efficiency of selected models for each research sample [%]

Year of birth	No. of ratios	Sample	AS (424)	AB (600)	AB(10 000)	Average
		Model	TE	EI	EII	TE
1996	5	Gajdka, Stos 2	75.5	66.1	64.8	68.8
2005	4	Prusak 4	75.7	72.8	53.0	67.1
2005	3	Prusak 3	73.9	63.9	61.7	66.5
1987	3	Legault, Score	73.0	53.8	67.3	64.7
1978	4	Springate	72.9	62.7	57.6	64.4

Year of birth – the year in which a model was constructed.
EI – type I efficiency, EII – type II efficiency, TE – total efficiency.
Source: Authors' work.

Table 5. Analysis of the efficiency of selected models
for particular research samples by rank

Sample	AS (424)	AB (600)	AB (10 000)	A
Model	TE	E I	E II	TE
Prusak 4	1	2	26	3.73
Gajdka, Stos 2	2	4	22	5.60
Prusak 3	3	5	24	7.11
Springate	5	6	25	9.09
Legault, Score	4	9	21	9.11

AS – average for the small sample, AB – average for the large sample, A – average
EI – type I efficiency, EII – type II efficiency, TE- total efficiency.
Source: Authors' work.

Secondly, the models were compared on the basis of the large sample according to type I efficiency (Table 2) and type II efficiency (Table 3). Once again, the Prusak 4 model achieves the highest rate of detecting enterprises that go bankrupt. The type I efficiency of this model is 73%. In addition to this model, in the forefront of the most effective models are: the Gajdka, Stos 2 model (66%), the Prusak 3 model (64%) and the Springate model (63%). However, they achieve considerably lower correct classification rates in comparison to the most efficient model. The other models do not classify enterprises sufficiently well.

The Legault–Score model is also characterized by the highest type II efficiency for correctly classifying businesses that remain in operation in the large sample. It should be noted that models with the highest type II efficiency, i.e., recognize businesses which have a good financial condition well, are less efficient at recognizing firms with poor financial standing. Similarly, models which are characterized by high type I efficiency tend to achieve low type II efficiency. Such properties indicate the poor overall performance of, e.g., the Legault–Score model. A collective analysis of models is presented in Table 4.

Analysing the results of the overall comparison of the selected models for the individual test samples, it should be pointed out that the highest overall efficiency rate is achieved by the Gajdka, Stos 2 model – almost 69%. In addition, four models achieve a similar overall accuracy for the classification of businesses. These models are the following: Prusak 4, Legault, Prusak 3 and Springate. These four models are ones specifically developed for the manufacturing sector. Two of them are models which were constructed for companies operating in Canada. Therefore, these models can be used as a baseline against which to assess the efficiency of Polish models. In addition to the Gajdka, Stos 2 model, models which are specifically designed for industry achieve higher overall efficiency. This statement is the answer to the fourth research question.

Table 6. The financial indicators used in the top 5 among 33 discriminant models

Ratio	Prusak 4	Gajdka, Stos 2	Legault, Score	Prusak 3	Springate
$X1 = \frac{profit\_on\_sales}{total\_sales}$	yes			yes	
$X2 = \frac{operating\_expenses}{short\text{-}term\_liabilities}$	yes			yes	
$X3 = \frac{short\text{-}term\_liabilities}{total\_assets}$	yes				
$X4 = \frac{profit\_on\_operating\_activities}{total\_assets}$	yes				
$X5 = \frac{(current\_liabilities \times 365)}{cost\_of\_products\_sold}$		yes			
$X6 = \frac{net\_profit}{total\_assets}$		yes			
$X7 = \frac{gross\_profit}{sales}$		yes			
$X8 = \frac{total\_assets}{total\_liabilities}$		yes			
$X9 = \frac{equity}{total\_assets}$			yes		
$X10 = \frac{gross\_profit + extraordinary\_items}{total\_assets} + \frac{financial\_expenses}{total\_assets}$			yes		
$X11 = \frac{sales}{total\_assets}$			yes		yes
$X12 = \frac{current\_assets}{short\text{-}term\_liabilities}$				yes	
$X13 = \frac{working\_capital}{total\_assets}$					yes
$X14 = \frac{EBIT}{total\_assets}$					yes
$X15 = \frac{gross\_profit}{short\text{-}term\_liabilities}$					yes

Source: Author's work

In addition, increasing the number of indicators used in the model seems to have no positive effect on its overall efficiency. The most effective models are constructed on the basis of between three and five indicators. In contrast, the least efficient models consist of between five and eight indicators. The use of a larger number of indicators does not bring the desired effect of increasing the ability of a model to recognize the situation of a company. This is the answer to the third research question. What is more, the overall efficiency of the models constructed in the nineties of the twentieth century, apart from the Gajdka, Stos model, have significantly declined. Moreover, it was found that the overall performance of even some models created at the beginning of the twenty first century has also markedly declined. This is the answer to the second research question.

Table 6 shows the financial indicators that the five best models for classifying firms are based on. Fifteen indicators are used altogether, only two of which can be found in two different models. It should be added that many indicators appear repeatedly in various models of bankruptcy prediction [92]. The variable which appeared most commonly among the indicators from forty nine models considered in [91–93] is the current ratio, which also plays a role in the Prusak 3 model. Apart from that, there are other indicators which often appear, such as the equity ratio, debt ratio, assets turnover ratio, etc. The frequency of their appearance may testify to their predictive and discriminative ability [94], but these abilities may not be stable in time [90].

In Table 5 models were ranked according to the arithmetic mean of their highest efficiency in the various research tests. Hence, these results may not illustrate the true performance of the models, because each test differs according to the number of enterprises and the measure of a model's efficiency (overall, type I and type II efficiencies). Taking into account these differences, models were ranked according to their average rank in each set (the geometric mean is used here).

With regard to the ranks of models, the Prusak4 model was the most highly ranked of the models tested, above the Gajdka, Stos 2 model. However, to sum up the results obtained, no unique optimal model, according to which firms can be assessed with very high efficiency, has yet been established. This is the answer to the first, and most important, research question.

6. Discussion

Many models have been used to assess the financial condition of enterprises. The fact of the matter is that not every model efficiently recognizes the deteriorating financial standing of an enterprise early enough. Moreover, models should foresee bankruptcy relatively long in advance and only such models can be considered as useful.

The study shows that only 5 out of 33 models were characterized by sufficient predictive ability in the five years before the bankruptcy of an enterprise, but the differences between the effectiveness of these models are negligible.

Among the models developed outside Poland, the Altman model does not appear in the top five models, but the Springate model and the Legault–Score model do. These results are consistent with those of [3, 20, 44], who found that the efficiency of the Altman model has significantly decreased over time. However, in other studies, the Altman model and Springate model have performed better [28–30, 89, 99]. The varying effectiveness of the Altman model might be found in the changing predictive power of its predictors in different environments. The efficiency of this model may significantly decline when applying the model in emerging [2]. The need to re-estimate coefficients in these models might be another cause [63]. Leaving coefficients unaltered may cause a decline in the effectiveness of models, because models tend to lose their effectiveness with the passage of time from the moment of their construction[3]. The size of a research sample also affects the results obtained by models, especially if the sample does not exceed 24 companies [29], 32 companies [30], or 60 companies [65]. This constitutes a certain limitation and therefore the relatively high efficiencies obtained in our study must be looked at with a critical eye, because these results may be sensitive to the sample size and the period of time [3].

Besides the analysis of models developed outside Poland, the results were compared to the latest published results regarding Polish models [1, 33, 55]. The study period is very similar to the period covered by the present research: 2010–2013, 2009–2012 and 2006–2012 (3 years before bankruptcy), respectively. However, the size of the samples is much smaller than in the case of the present research: a single company, 110 companies and 142 companies. The results of Kisielińska [55] and Adamowicz and Noga [1] are similar, especially regarding the effectiveness of classification by the Poznan model (82.7% and 82%, respectively), but these figures are much different to the effectiveness obtained in our study, 58.0%. There might be many reasons for these dissimilarities. Some of them were mentioned above.

On the basis of Kopczyński's research, in practice, companies operating in Poland do not use advanced tools such as univariate and multivariate models, to predict bankruptcy. One reason for this is a lack of knowledge regarding such methods. Even if firms have heard about them, they are not able to apply them [59]. Therefore, researchers should take action to popularize models of bankruptcy prediction and educate managers how to use them.

The limited acceptance by managers of discriminant models also results from the inherent imperfections in the concept of such models:

[3] Therefore, re-estimation of the coefficients in the models analysed seems to be reasonable. Also, in future research, these models will be further developed and re-analysed.

• There is no single theoretical basis on which various econometric schools propose different sets of indicators. Hence, their personal, sometimes arbitrary, judgment is applied when estimating the risk of bankruptcy.

• To determine the parameters of the discriminant model, one must have a properly chosen test sample, in which both companies with a good financial position and those that have gone bankrupt are represented.

• The business environments of these companies must be comparable in the long term, which eliminates the application of such methods to companies that operate in fragile economies, e.g., those passing through a period of fundamental system transformation, as in the case of Poland at the end of the 20th century.

Also, the method does not take into account the impact of overall economic prosperity on the economic condition of a company and the specificity of a given branch of industry. Consequently, more and more often – as an alternative to discriminant models – simulation models and techniques derived from artificial intelligence (neural networks) have been proposed as a tool for estimating the risk of a firm becoming insolvent.

7. Conclusions

This article presents a comparison of 33 discriminant models based on both results from the literature and analysing two research samples: one small and one large. The small sample was made up of 424 businesses. Using the large sample, the type I efficiency (based on nearly 600 enterprises) and the type II efficiency (on the basis of more than 10 thousand enterprises) of these models were analysed separately.

Models of bankruptcy prediction have been compared on the basis of these two research samples and the following answers to earlier posed research questions were obtained:

• The analysis of 33 models of bankruptcy prediction shows that only 5 models were characterized by sufficient predictive ability in the five years before the bankruptcy of enterprises.

• Most of the models built in the late twentieth century had significantly reduced overall efficiency. Also, even some models constructed in the early twenty first century evaluated the financial condition of businesses with low effectiveness.

• Increasing the number of indicators used in the model seems to have no positive effect on its overall performance. The most effective models are constructed on the basis of three to five indicators. On the other hand, the worst models consist of five to eight,

• The most effective models tend to be those specifically constructed for the manufacturing sector, except for the Gajdka, Stos 2 model, which is not specifically developed for industry.

Summing up, it must be stated that, so far, no single optimal model has been established, using which a very high efficiency can be assured. Therefore, it is necessary to continue research related to the construction of models for assessing the financial standing or corporations.

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